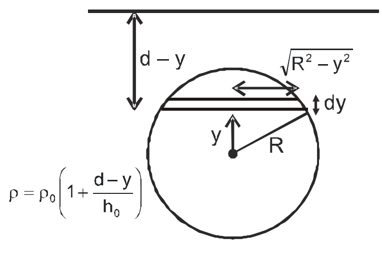
1. (1) In arrangement–1, water of weight ρVg gas come out, but the buoyancy force is also equal to the weight of displaced liquid. So, reading of weighing machine is W.
(2) In arrangement–2, weight of the ball mg is added, but water of weight $\rho_w Vg$ is removed so reading of weighing machine is W + mg – ρVg.

2. $dB = \pi(R^2 - y^2)dy \quad \rho_0\left(1+\frac{d-y}{h_0}\right)g$



$$dB = \frac{\pi\rho_0 g}{h_0}(R^2 - y^2)(h_0 + d - y)dy$$

$$= \frac{\pi\rho_0 g}{h_0}[R^2(h_0 + d)dy - R^2ydy - (h_0 + d)y^2dy + y^3dy]$$

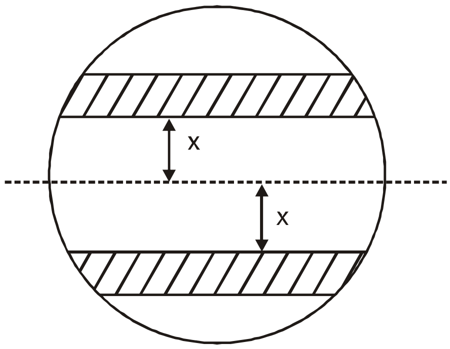
$$B = \int_{y=-R}^{+R} dB = \frac{\pi\rho_0 g}{h_0}\left(R^2(h_0+d)y - \frac{R^2y^2}{2} - (h_0+d)\frac{y^3}{3} + \frac{y^4}{4}\right)_{-R}^{+R}$$

$$B = \frac{\pi\rho_0 g}{h_0}\left[(h_0+d)R^2(2R) - \frac{(h_0+d)}{3}(2R^3)\right] \qquad = \frac{\pi\rho_0 g}{h_0}\left[\frac{4}{3}(h_0+d)R^3\right]$$

$$= \frac{4}{3}\pi R^3 g\frac{\rho_0}{h_0}(h_0+d) = \frac{4}{3}\pi R^3 g\sigma \quad \Rightarrow \quad \sigma = \frac{\rho_0}{h_0}(h_0+d)$$

$$\sigma = \rho_0\left(1+\frac{d}{h_0}\right)$$

Alternate solution



$$\sigma vg = \int\left[\rho_0\left(1+\frac{d-x}{h_0}\right)dvg + \rho_0\left(1+\frac{d+x}{h_0}\right)dvg\right]$$

$$\sigma v = 2\rho_0\left(1+\frac{d}{h_0}\right)\int_0^{v/2} dV = \rho_0 v\left(1+\frac{d}{h_0}\right)$$

$$\Rightarrow \qquad \sigma = \rho_0\left(1+\frac{d}{h_0}\right)$$

3. From Fig.(a) h_2A = volume of oil + some volume of ice

From Fig. (b) $h_2'A$ = volume of oil

$\Rightarrow$ $(h_2 - h_2')A$ = some volume of ice > 0

$\Rightarrow$ $h_2 > h_2'$

$\therefore$ Statement 3 correct

Pressure at bottom in fig. (a), is given by

$\Rightarrow$ $P_0 + \rho_{oil}\ h_2g + \rho_{water}\ h_1g$

$\therefore$ $(P_0 + \rho_{oil}\ h_2g + \rho_{water}h_1g)\ A = P_0A + W_{oil} + W_{water} + W_{ice}$ (i)

Similarly from fig. (b)

$(P_0 + \rho_{oil}\ h_2'g + \rho_{water}h_1'g)A = P_0A + W_{oil} + W_{water} + W_{ice}$ (ii)

$\rho_{oil}\ h_2' + \rho_{water}\ h_1' = \rho_{oil}\ h_2 + \rho_{water}\ h_1$

$\Rightarrow$ $\rho_{oil}(h_2 - h_2') = \rho_{water}(h_1' - h_1)$

$\Rightarrow$ $h_1' - h_1 = \dfrac{\rho_{oil}}{\rho_{water}}(h_2 - h_2') > 0$

$\therefore$ Statement 2 is correct.

Now fall in level $= |h_2 - h_2'|$

and rise in level $= |h_1' - h_1|$

$= \dfrac{\rho_{oil}}{\rho_{water}}(h_2 - h_2') < h_2 - h_2')$

$\Rightarrow$ Fall is more

Statement b is correct

4. $-A\dfrac{dy}{dt} = a\sqrt{2gy}$

$$\frac{2A}{a\sqrt{2g}}\left(\sqrt{H} - \sqrt{\frac{H}{n}}\right) = T_1$$

$$\frac{2A}{a\sqrt{2g}}\left(\sqrt{\frac{H}{n}} - 0\right) = T_2$$

$T_1 = T_2$

n = 4.

5. Upward force by capillary tube on top surface of liquid is

$f_{up} = 4\sigma a \cos\theta$

If liquid is raised to a height h then we use

$4\sigma a \cos\theta = ha^2\rho g$ or $h = \dfrac{4\sigma\cos\theta}{a\rho g}$ **Ans.**

6. The only force acting on the body is the viscous force

Here $m\frac{vdv}{dx} = -6\pi\eta rv \quad = -rv$

$\Rightarrow \int_v^0 mdv = \int_0^x -rdx \qquad \Rightarrow x = \frac{mv}{r}$.

7. From the free body diagram of the sphere :

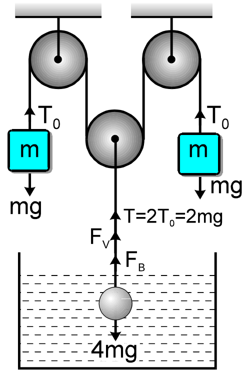
$F_V = 4\,mg - 2\,mg - F_B$

$\Rightarrow \quad F_V = 2\,mg - F_B$

$\Rightarrow \quad 6\pi\eta r V = \frac{4}{3}\pi r^3\left(\frac{\sigma}{2} - \rho\right)g$

(since $4m = \frac{4}{3}\pi r^3 \times \sigma$)

$\Rightarrow \quad V = \frac{1}{9}\frac{r^2(\sigma - 2\rho)g}{\eta}$



8. Let the density of water be ρ, then the force by escaping liquid on container = $\rho S(\sqrt{2gh})^2$

$\therefore$ acceleration of container a = $\frac{2\rho Sgh - \mu\rho Vg}{\rho V} = \left(\frac{2Sh}{V} - \mu\right)g$

Now $\mu = \frac{Sh}{V} \qquad \therefore a = \frac{Sh}{V}g$

9. Viscous force = mg sin θ

$\therefore \eta A\,\frac{v}{t} = mg\sin\theta \quad$ or $\quad \eta a^2\,\frac{v}{t} = a^3\rho g\sin\theta$

$\eta = \frac{t\rho g\sin\theta a}{v}$

10. Relative to liquid, the velocity of sphere is $2v_0$ upwards.

$\therefore$ viscous force on sphere $= 6\pi\eta r\,2v_0$ downward

$= 12\pi\eta r v_0$ downward

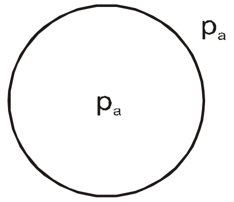
11. The force exerted by film on wire or thread depends only on the nature of material of the film and not on its surface area. Hence the radius of circle formed by elastic thread does not change.

12. (B) Inside pressure must be $\frac{4T}{r}$ greater than outside pressure in bubble. This excess pressure is provided by charge on bubble.

$\frac{4T}{r} = \frac{\sigma^2}{2\varepsilon_0}$

$\frac{4T}{r} = \frac{Q^2}{16\pi^2 r^4 \times 2\varepsilon_0} \qquad \ldots\ldots\left[\sigma = \frac{Q}{4\pi r^2}\right]$

$Q = 8\pi r\sqrt{2rT\varepsilon_0}$



13. Isothermal process.

$$\left(P_1+\frac{4T}{r}\right)\left(\frac{4}{3}\pi r^3\right)=\left(P_2+\frac{4T}{r/2}\right)\left(\frac{4}{3}\pi (r/2)^3\right)$$

$$P_2 = 8P_1 + \frac{24T}{r}$$

14. Given :
Initial radius of soap bubble = R
Surface tension of soap solution = T
Final radius of soap bubble = 2R
The initial energy needed to blow the soap bubble is
$E_1 = 2 \times 4\pi R^2 \times T = 8\pi R^2 T$
and final energy needed to blow the soap bubble is
$E_2 = 2 \times 4\pi (4R)^2 = 32\pi R^2T$
Hence extra energy is needed is given by
$E_2 - E_1 = 32\pi R^2T - 8\pi R^2T = 24\pi R^2T$

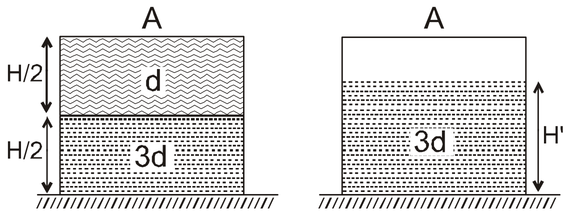
15. Let v be the velocity of the movable plate and F is equal to viscous force

$$F = \left[\eta_1 \frac{v}{h_1} + \eta_2 \frac{v}{h-h_1}\right]A \Rightarrow \frac{dF}{dh_1} = 0 \qquad \therefore h_1 = \frac{h}{3}$$

16. A, B, D

$$\frac{H}{2}\times d + \frac{H}{2}\times 3d = H' \times 3d$$

$$\Rightarrow H' = \frac{2H}{3}$$



$V_{efflux} = \sqrt{2g(H'-h)}$

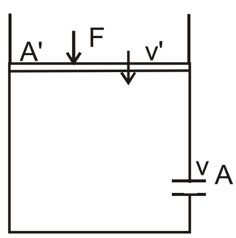
V_{efflux} is maximum when h = H'/2

$$\therefore \quad V_{max} = \sqrt{\frac{2gH}{3}}$$

Range $R = V_{efflux} \times \sqrt{\frac{2(H'-h)}{g}}$

$$R_{max} = \frac{2H}{3}$$

17. $\frac{F}{A} + \frac{1}{2}\rho v'^2 = \frac{1}{2}\rho v^2$ (i)



$A'v' = Av$ (ii)

$\therefore F \propto v^2$ (A)

$P = F \cdot v'$ (B)

Av = volume flow rate $= \frac{\text{volume}}{t}$

$\therefore t \propto \frac{1}{v}$ (C)

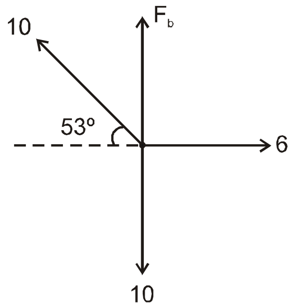
W.D. = ΔK $\Rightarrow$ (D)

18. $F_{drag} = 6\pi\eta RV$

$= 6\pi \dfrac{20}{6\pi} \times 0.1 \times 5 = 10$ N

$F_b + 8 = 10$

$F_b = 2$

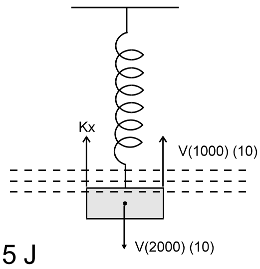


19. $Kx = V(2000)(10) - V(1000)(10)$

$= \dfrac{10}{2000}[1000 \times 10]$

$Kx = 50$ N ... (b)

$U_{stored} = \dfrac{1}{2} \times (100)\left(\dfrac{50}{100}\right)^2 = \dfrac{1}{2} \times \dfrac{2500}{100} = 12.5$ J



20. S = 0.5 N/m $r = 10^{-3}$ m $\theta_C = 120°$ $\rho = 5 \times 10^3$ kg/m^3

$h_{,max} = \dfrac{2S\cos\theta_c}{r\rho g} = \dfrac{(2)\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{(10^{-3})(5\times10^3)(10)} = 10^{-2}$ m = 1cm

If $h = \dfrac{h_{max}}{2}$

$\dfrac{2S\cos\theta}{r\rho g} = \dfrac{1}{2}\dfrac{2S\cos\theta_c}{r\rho g}$

$\Rightarrow \cos\theta = -\dfrac{1}{4}$

$\theta = \cos^{-1}\left(-\dfrac{1}{4}\right)$

If $h = \dfrac{h_{max}}{3}$

$\dfrac{2S\cos\theta}{r\rho g} = \dfrac{1}{3}\dfrac{2S\cos\theta_c}{r\rho g}$

$\Rightarrow \cos\theta = -\dfrac{1}{6}$, $\theta = \cos^{-1}\left(-\dfrac{1}{6}\right)$

21. $h = \dfrac{2T\cos\theta}{\rho gr}$

22. $\rho gh = 1/2\ \rho v_1^2$ ....(1)

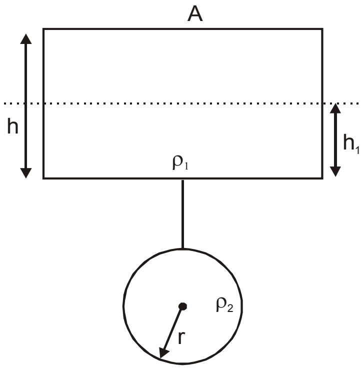
$\Delta P = \rho gh = 1/2\ \rho v_2^2$ ....(1)

$v_2 = 3v_1 \Rightarrow v_2^2 = 9v_1^2$

$\Rightarrow \dfrac{1}{2}\rho v_2^2 = 9\left(\dfrac{1}{2}\rho v_1^2\right) \Rightarrow \Delta P + \rho gh = 9\rho gh$

$\Delta P = 8\rho gh = 8 \times 10^3 \times 10 \times 10 = 8 \times 10^5$ pascal = 8 atm

23. Taking cylinder and the ball as system



$$\frac{4}{3}\pi R^3 . \rho_2 . g + Ah.\rho_1 g = \frac{4}{3}\pi R^3 . \rho_w . g + Ah_1.\rho_w g$$

$$\rightarrow R = \left[\frac{3A(h_1\rho_\omega - h\rho_1)}{4\pi(\rho_2 - \rho_w)}\right]^{1/3}$$

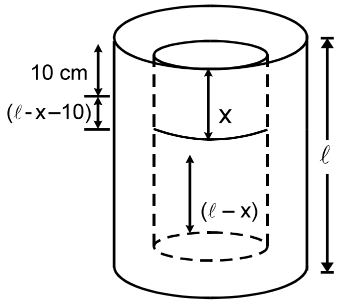
using values

A = 11 cm^2 ; h_1 = 4 cm ; ρ_w = 1 gm/cm^3 ;

ρ_1 = 0.5 gm/cm^3 ; ρ_2 = 8gm/cm^3

$$R = \left[\frac{3\times 11(4\times 1 - 6\times 0.5)}{4\times\left(\frac{22}{7}\right)\times(8-1)}\right]^{1/3} = \left(\frac{3}{8}\right)^{1/3}\text{cm} \quad \Rightarrow \quad R^3 = 3/8$$

24. After oil is filled up, pressure at the depth of lower end should equate if measured from inside and outside the tube. Suppose depth of oil is x cm then :



1000.g. [(ℓ – 10)cm] = 800 g. (x cm) + 1000 g[(ℓ – x)cm] $\Rightarrow$ x = 50 cm

25. The coefficient of viscosity is the ratio of tangential stress on top surface of film (exerted by block) to that of velocity gradient(vertically downwards) of film. Since mass m moves with constant velocity, the string exerts a force equal to mg on plate towards right. Hence oil shall exert tangential force mg on plate towards left.

$$\therefore \ \eta = \frac{F/A}{(v-0)/\Delta x} = \frac{125\times 1000/10\times 20}{(5-0)/.02} = 2.5 \text{ dyne–s/cm}^2$$

26. Magnitude of viscous force, $F = \eta A \frac{dv}{dr}$

$\Rightarrow$ viscous force per unit area $\sigma = \frac{F}{A} = \eta\frac{dv}{dr}$

$$v = v_0\left(1-\frac{r^2}{R^2}\right) \Rightarrow \frac{dv}{dr} = -\frac{2V_0 r}{R^2} \Rightarrow \sigma = \eta\cdot\frac{2v_0 r}{R^2} \quad ......(i)$$

Volume rate of flow, Q

consider an annular element at r from axis, width dr.

$$dA = 2\pi r dr \ ; \ dQ = v.dA = v_0\left(1-\frac{r^2}{R^2}\right)2\pi r dr$$

$$Q = \int dQ = 2\pi v_0\left[\frac{r^2}{2}-\frac{r^4}{4R^2}\right]_0^R = \frac{\pi}{2}R^2 v_0 \quad \Rightarrow \quad v_0 = \frac{2Q}{\pi R^2}$$

$$\therefore \quad (i) \quad \Rightarrow \quad \sigma = \eta\frac{4Q}{\pi R^4} r, \ R = 0.1 \text{ m}$$

$$\text{At } r = 0.04 \text{ m}, \ \sigma = (0.75)\, 4 \times \frac{\pi}{2} \times 10^{-2} \times \frac{0.04}{\pi\times 10^{-4}} = 6 \text{ Nm}^{-2}$$

27. The F.B.D. of wire PQ is

The force due to surface tension = F_{ST} = 2T × 2 AD tanθ

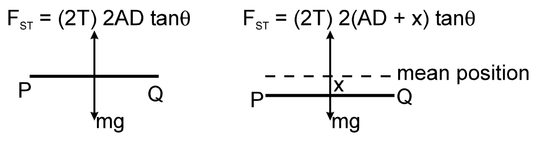


Figure (a) Figure (b)

For wire to be in equilibrium (Figure (a))

4T AD tanθ = mg .... (1)

If the wire PQ is at a distance x below the mean position, the restoring force on the wire is (Figure (b))

– ma = 4T tanθ (AD + x) – mg = 4T tanθ x

Hence the wire PQ executes SHM

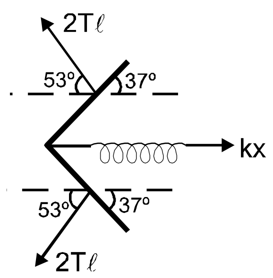
$$a = -\frac{4T}{m}\tan\theta \quad x$$

comparing with $a = -\omega^2 x$ we get

$$\omega^2 = \frac{4T}{m}\tan\theta$$

or $T = 2\pi\sqrt{\frac{m}{4T\tan\theta}} = 2\pi\sqrt{\frac{1\times10^{-3}}{4\times25\times10^{-3}}} = \frac{\pi}{5}$ s

28.



2(2Tℓ) cos53° = Kx

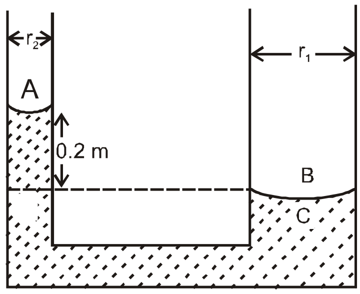
$$\frac{4T\ell 3}{5K} = x.$$

29. $F_d = 6\pi\mu r u$

$F_B = \frac{4}{3}\pi r^3\sigma g$, $mg = \frac{4}{3}\pi r^3 \rho g$

$mg - F_d - F_B = ma$; $u_0 = \frac{2r^2}{9}g\frac{(\rho-\sigma)}{\mu}$

$\therefore \quad a = \left(1-\frac{\sigma}{\rho}\right)\left(1-\frac{u}{u_0}\right)g$

30. 

$r_1 = 1.44 \times 10^{-3}$ m. $r_2 = 0.72 \times 10^{-3}$ m.

Equating pressures at points (B) & (C)

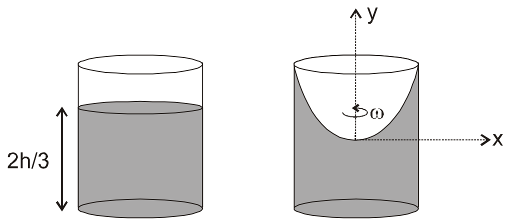
$P_A - \frac{2\sigma}{r_2} + (0.2)\,\rho g = P_C.$ and $P_B - \frac{2\sigma}{r_1} = P_C.$

so $P_B - P_A = 2\sigma\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + 0.2\,\rho g$

$= 2 \times 72 \times 10^{-3}\ \frac{N}{m}\left[\frac{10^3}{1.44} - \frac{10^3}{0.72}\right] + (0.2) \times 10^3 \times 938$

$= \frac{144 \times (-0.72)}{1.44 \times 0.72} + 1960 = -100 + 1960 = 1860$ N/m².

31. Profile of rotating liquid is given by



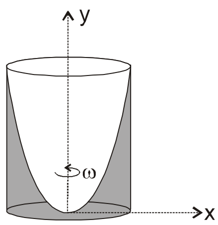
$$y = \frac{\omega^2 x^2}{2g}$$

Putting x = a, $y = \frac{\omega^2 a^2}{2g}$

Volume of liquid in fig. (b) is written as $= \frac{\pi a^2 \times \frac{\omega^2 a^2}{2g}}{2} + \pi a^2\left(h - \frac{\omega^2 a^2}{2g}\right)$

Equating to volume in figure (a), we get

$$\pi a^2 \frac{2h}{3} = \frac{\pi a^2 \omega^2 a^2}{4g} + \pi a^2\left[h - \frac{\omega^2 a^2}{2g}\right] \Rightarrow \omega = \sqrt{\frac{4gh}{3a^2}}$$

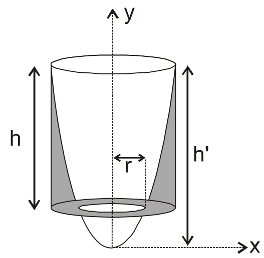


32. $y = \dfrac{\omega^2 x^2}{2g}$

$h = \dfrac{\omega^2 a^2}{2g}$

$\omega = \sqrt{\dfrac{2gh}{a^2}}$

33. Some base area will be visible. Let radius of visible base be 'r'.
Origin shifts below base.
Put x = a & y = h'



$$h' = \frac{\omega^2 x^2}{2g} = \frac{8gh}{a^2} \times \frac{a^2}{2g} = 4h$$

put x = r & y = h' – h = 3h

$$3h = \frac{8gh}{a^2} \times \frac{r^2}{2g} \quad \Rightarrow \quad r^2 = \frac{3}{4}a^2$$

$$V_{left} = \frac{\pi a^2 h^1}{2} - \left[\pi(a^2 - r^2)(h' - h) + \frac{\pi r^2 (h' - h)}{2}\right] \qquad = \frac{\pi a^2 h}{8}$$

34 to 36

(i) By conservation of volume

$4 \times h = 4 \times 2 + 2 \times 1 = 10$

$h = 2.5m$

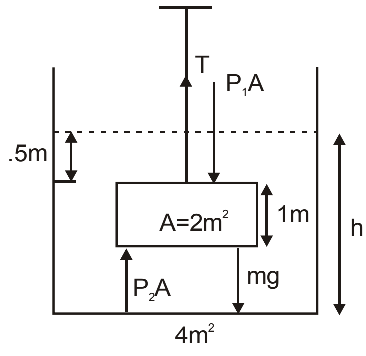
Pressure at top of the object

$= P_0 + 0.5 \times 1000 \times 10$

$= 1.05 \times 10^5$ N/m^2

$F = P_1 A$

$= 1.05 \times 10^5 \times 2 = 2.1 \times 10^5$ N



By F.B.D. $T + P_2 A = mg = P_1 A$

$T = mg + (P_1 - P_2) A$

$= mg - (P_2 - P_1) A$

$= 2 \times 2000 \times 10 - (.2 \times 10^5)$

$= .4 \times 10^5 - 0.2 \times 10^5 = 0.2 \times 10^5$ N

$F_b = V.\rho_w g$

$= 2 \times 1000 \times 10 = 0.2 \times 10^5$ N

It is also equal to net contact force by the liquid $= P_2 A - P_1 A$

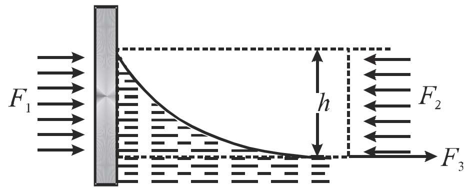
$= 0.2 \times 10^5$N

Note : Net contact force and buoyant force are same.

37-39

The pressure of the water changes linearly with the increase in height. At the bottom of the meniscus it is equal to the external atmospheric pressure p_0, and at the top to . The average pressure exerted on the wall is $p_{average} = p_0 - \rho gh/2$. The force corresponding to this value, for an aquarium with side walls of length ℓ, is $F_1 = p_{average}\ell h$.

Consider the horizontal forces acting on the volume of water enclosed by the dashed lines in the figure. The wall pushes it to the right with force F_1, the external air pushes it to the left with force $F_2\ \pi_0\ell h$, and the surface tension of the rest of the water pulls it to the right with a force $F_3 = \ell s$. The resultant of these forces has to be zero, since the volume itself is at rest. This means that



$$\left(p_0 - \frac{1}{2}\rho gh\right)\ell h - p_0\ell h + \ell s = 0,$$

which we can write as

$$h = \sqrt{\frac{2s}{\rho g}} = \sqrt{\frac{2\times 0.073}{1000\times 10}} = 0.0038 \text{ m}.$$

water rises by approximately 4 mm up the wall of the aquarium.

$$\left(p_0 - \frac{1}{2}\rho gh\right)\ell h - p_0\ell h + \ell s = 0,$$

which we can write as $h = \sqrt{\frac{2s}{\rho g}} = \sqrt{\frac{2\times 0.073}{1000\times 10}} = 0.0038 \text{ m}$

40 to 42

For this type of parallel flow the shearing stress is given as

$$\tau = \eta\frac{du}{dy} \qquad \ldots(i)$$

For the given distribution

$$\frac{du}{dy} = -\frac{3Vy}{h^2} \qquad \ldots(ii)$$

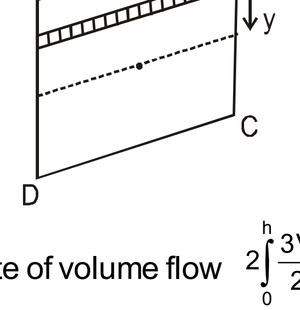
(a) Along the bottom wall so that (from eq. ii)

$\frac{du}{dy} = \frac{3V}{h}$ and therefore the shearing stress is $\tau_{\substack{bottom\\wall}} = \eta\left(\frac{3V}{h}\right)$

(b) Along the plane where y = h/2 it follows from equation (ii)that

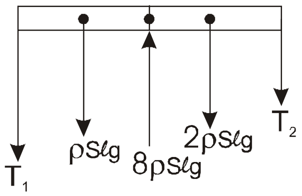
$\frac{du}{dy} = \frac{-3Vy}{h^2}$ and thus the shearing stress is $|\tau| = \eta\frac{3V}{2h}$.

(C)



Rate of volume flow $2\int_0^h \frac{3V}{2}\left(1 - \frac{y^2}{h^2}\right)dy.\ell = 2V\ell h$

43. Consider the FBD shown in the figure.



Force balance

$\Rightarrow \quad T_1 + T_2 = 5\rho S\ell g$

Torque balance about point P

$\Rightarrow \quad T_1 \times 2\ell + \rho S\ell g \times 3/2\ell - 8\rho S\ell g \times \ell/2 = 0$

$T_1 = 11/4\,\rho S\ell g$

$\Rightarrow \quad T_2 = 9/4\rho S\ell g$

$g_{eff} = g - g/2 = g/2$

44. $F = mA = F_0 - F_v = F_0 - \dfrac{\eta A v}{d}$

$A = \dfrac{F_0}{m} - \dfrac{\eta A}{md}\, v = a - bv$

$\dfrac{dv}{dt} = a - b\,v \quad \Rightarrow \quad \int_0^v \dfrac{dv}{a - bv} = \int_0^t dt$

$\Rightarrow \left(-\dfrac{1}{b}\right) \ell n \left(\dfrac{a - bv}{a}\right) = t \Rightarrow V = \dfrac{a}{b}\,(1 - e^{-bt})$

$\dfrac{dx}{dt} = \dfrac{a}{b}\,(1 - e^{-bt}) \Rightarrow \int_0^x dx = \dfrac{a}{b}\int_0^t (1 - e^{-bt})\,dt$

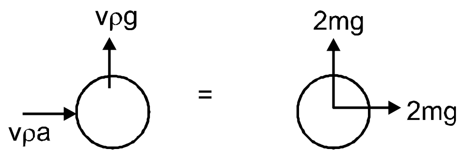
$x = \dfrac{a}{b}t - \dfrac{a}{b^2} + \dfrac{a}{b^2}e^{-bt}$

$A = ae^{-bt}$

$k = \dfrac{1}{2}\,mv^2$

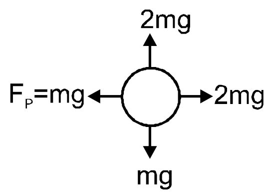
$\dfrac{dk}{dt} = m\,v\,\dfrac{dv}{dt} = \dfrac{ma}{b}\,(1 - e^{-bt})\,(ae^{-bt}) = \dfrac{ma^2}{b}\,(e^{-bt} - e^{-2bt})$

45. **(P)**



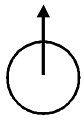
$F_B = \sqrt{2}\,(2mg)$

w.r.t. vessel

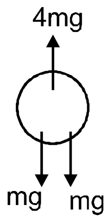


(P) – 1,3,4

(Q) $F_B = v\rho(g + g) = 4mg$



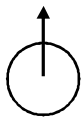
w.r.t. vessel



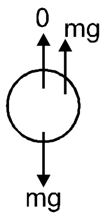
(Q) – 3,4

(R) $F_B = v\rho(g - g) = 0$

w.r.t. vessel

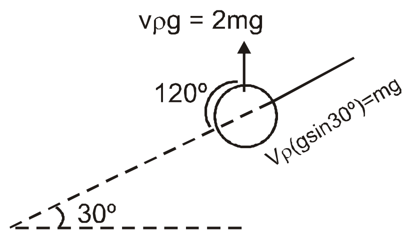


w.r.t. vessel



(R) – 2

(S) $F_B = v\rho g = 2mg$



$F_B = v\rho g = 2mg$

$$F_B = \sqrt{(2mg)^2 + (mg)^2 - 2mg \times mg \times 2 \times \frac{1}{2}}$$

$F_B = \sqrt{3}\,mg$

$$F_R = \sqrt{(mg)^2 + (mg)^2 + 2(mg)(mg) \times \frac{1}{2}}$$

$F_R = \sqrt{3}\,mg$

(S) – 3,4